EMI – part II

(mutual and self inductance)

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Inductance

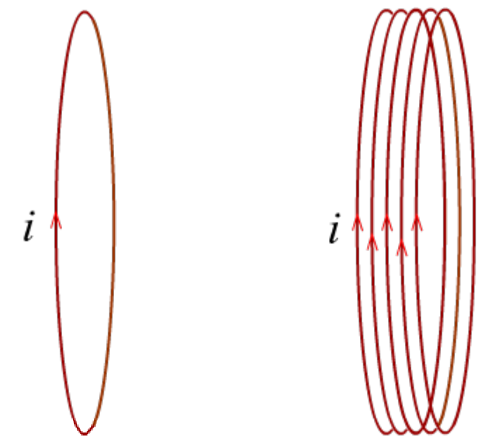
Magnetic flux linked to each turn of closed coil is directly proportional to the current.

$$\phi \propto i$$

For a coil of N turns and whose geometry remains constant, variation in flux occurs only due to variation of current, therefore

$$N\phi \propto i$$

$$N\phi = \text{const} \times i$$

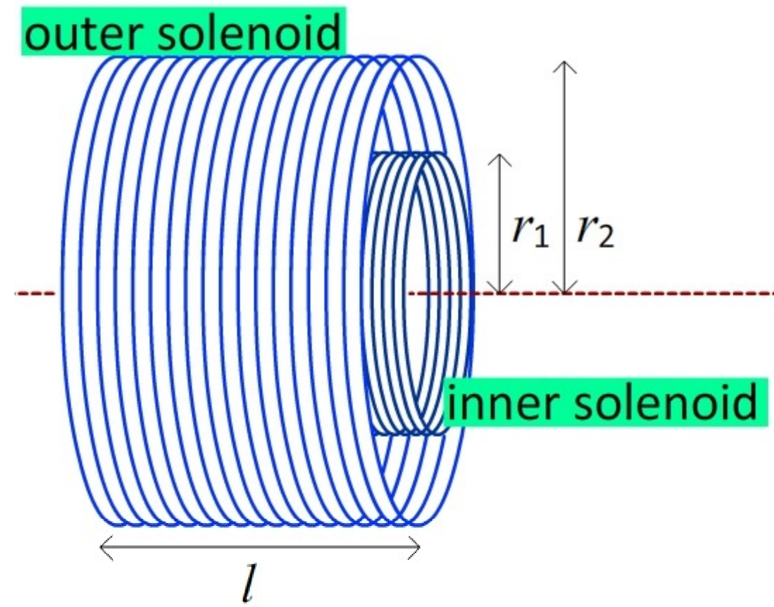


closed loop 1 turn   closed loop N turns

The constant of proportionality is called <u>inductance</u>. It depends only on the geometry of the coil and the medium.
SI unit of inductance is henry (H)

Mutual inductance (M)

Mutual induction is a phenomenon in which changing current in one coil induces emf in another coil linked to it.



Consider two long coaxial solenoids as shown in the figure.

Change in current through one of the solenoids results in a time varying flux linked to the other solenoid. This results in mutual induction.

Flux linked with any one of the coils depends on the geometry of the setup such as length, radius, number of turns per unit length etc and this determines the mutual induction between the coils.

n_1	number of turns per unit length of inner solenoid
n_2	number of turns per unit length of outer solenoid
l	length of the solenoid
r_1	radius of inner solenoid
r_2	radius of outer solenoid

Mutual inductance (M)

Case (I)

Current flowing through the inner coil

Let the current through the inner coil be i

Magnetic field due to the current is given by (neglecting fringe effects)

$$B = \mu_o n_1 i$$

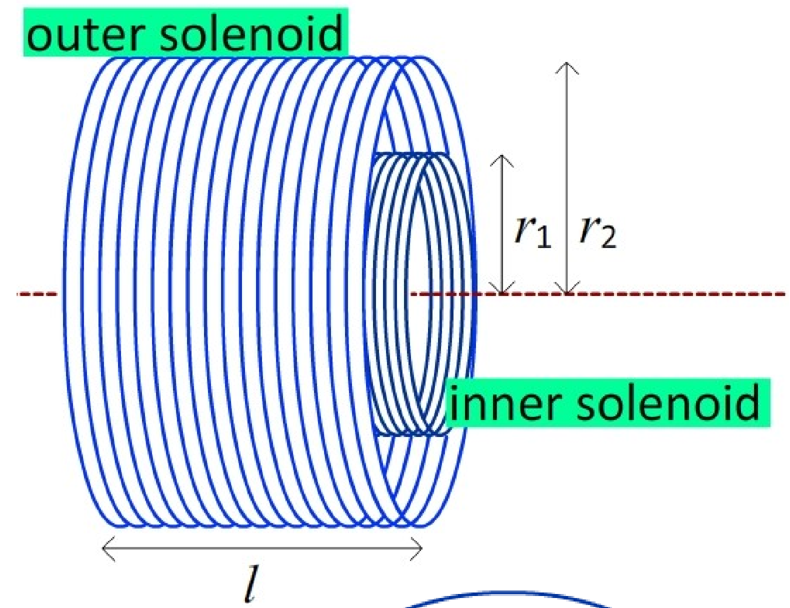
Flux linked with the outer coil (of N_2 turns) is given by

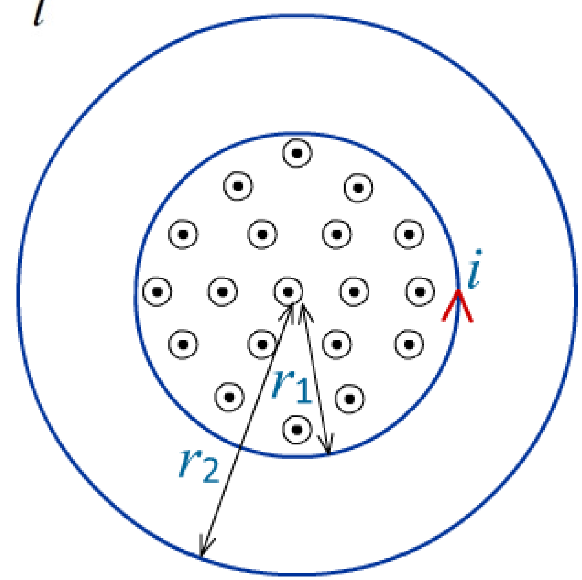
$$N_2\phi_2 = \mu_o n_1 i \times n_2 l \pi r_1^{\,2}$$

$$\Rightarrow \quad N_2\phi_2 = (\mu_o n_1 n_2 l \pi r_1^{\,2}) \times i$$

Mutual inductance of the pair of coils is therefore given by

$$\boxed{M = \mu_o n_1 n_2 l \pi r_1^{\,2}}$$





Cross-section view

Mutual inductance (M)

Case (II)
Current flowing through the outer coil

Let the current through the inner coil be i
Magnetic field due to the current is given by (neglecting fringe effects)

$$B = \mu_o n_2 i$$

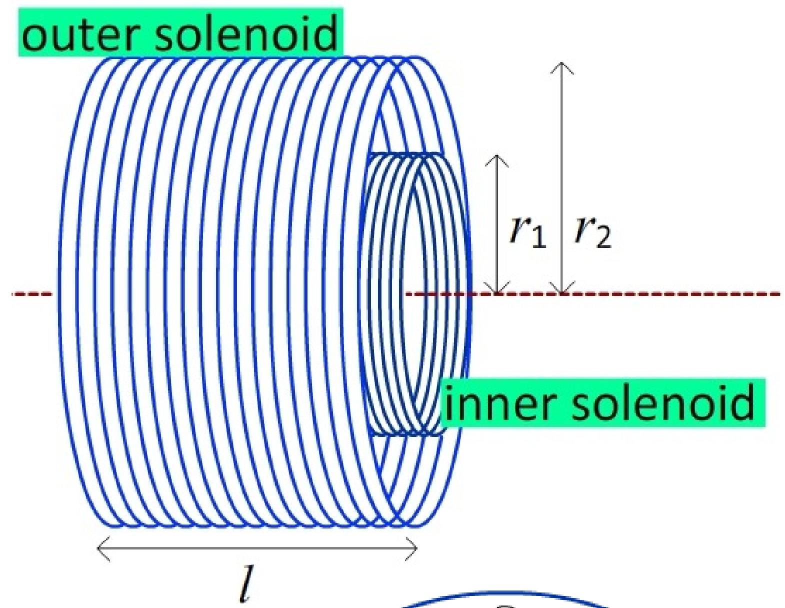
Flux linked with the inner coil (of N_1 turns) is given by

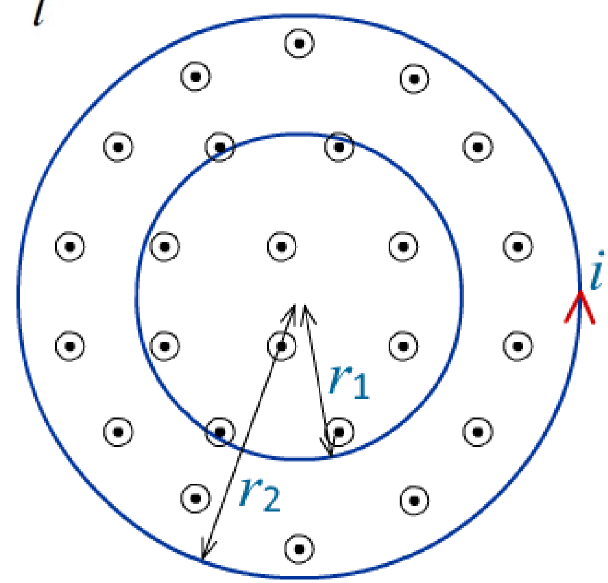
$$N_1\phi_1 = \mu_o n_2 i \times n_1 l \pi r_1^{\,2}$$

$$\Rightarrow \quad N_1\phi_1 = (\mu_o n_1 n_2 l \pi r_1^{\,2}) \times i$$

Mutual inductance of the pair of coils is therefore given by

$$\boxed{M = \mu_o n_1 n_2 l \pi r_1^{\,2}}$$

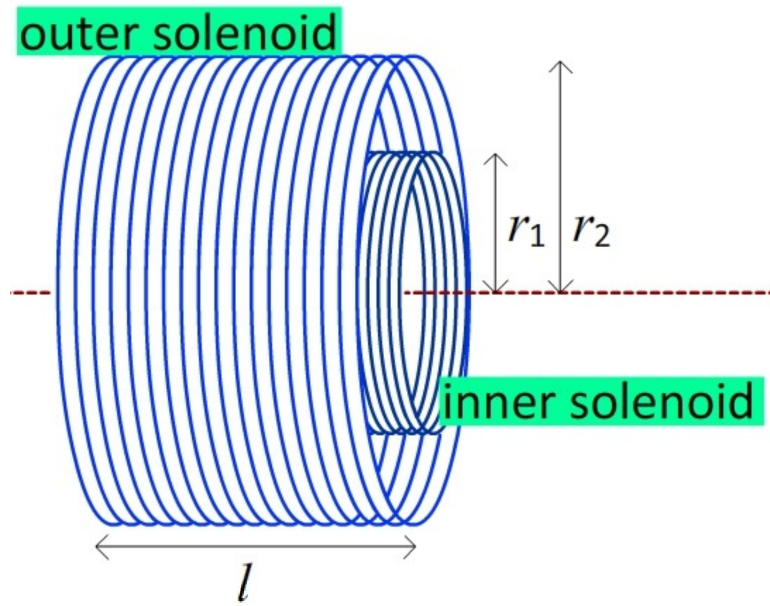




Cross-section view

Mutual induction of a pair of coaxially placed solenoids is

$$\boxed{M = \mu_o n_1 n_2 l \pi r_1^2}$$



Mutual inductance of a pair of coils is independent of the coils carrying the current. It depends only on the geometry of the setup leading to mutual induction

$$\boxed{\phi_{\text{total}} = M\,i}$$

$$\boxed{\mathcal{E} = -M\frac{\mathrm{d}i}{\mathrm{d}t}}$$

SI unit of M : henry (H)

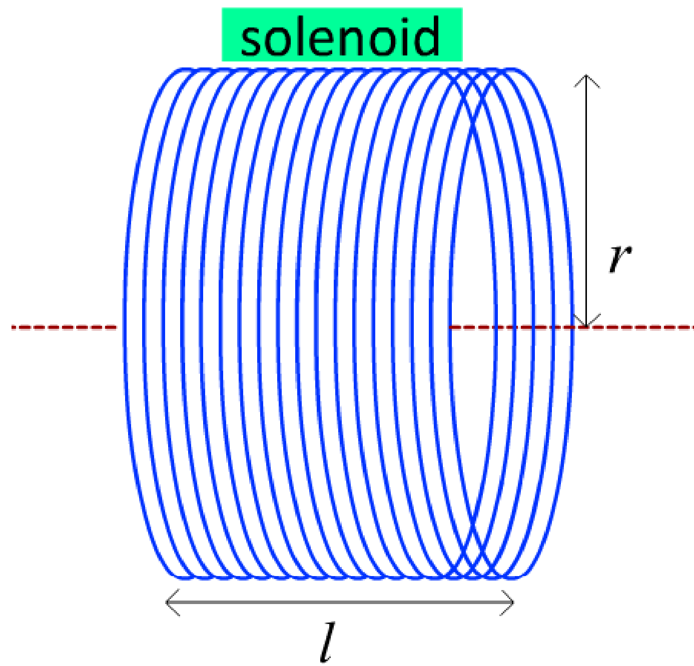
Mutual induction of a pair of coils is said to be one henry if a current of 1A in one coil links a flux of 1 weber in the other coil

Or

Mutual induction of a pair of coils is said to be one henry if current changing at a rate of $1As^{-1}$ in one coil results in an induced emf of 1V in the other coil

Self inductance (L)

Self induction is a phenomenon in which current changing in a coil induces emf in itself.



Let current in the solenoid be i.
Magnetic field due to the current is given by (neglecting fringe effects)

$$B = \mu_o n\, i$$

Flux linked with the solenoid (of N turns) is given by

$$N\phi = \mu_o ni \times nl\pi r^2$$

$$\Rightarrow \quad N\phi = (\mu_o nnl\pi r^2) \times i$$

Self inductance of the coil is therefore given by

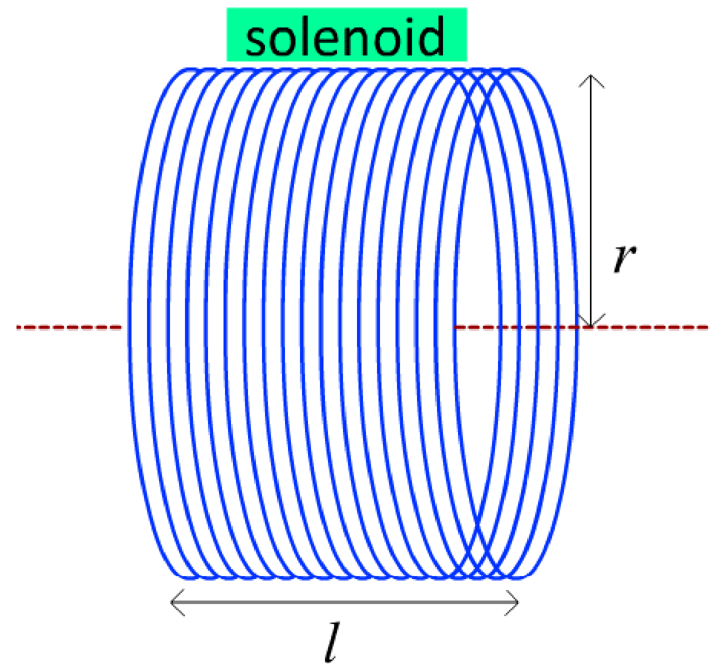
$$\boxed{L = \mu_o n^2 l\pi r^2}$$

n	number of turns per unit length of the solenoid
l	length of the solenoid
r	radius of the solenoid

Self inductance of a coil depends only on the geometry of the setup leading to self induction

$$\boxed{\phi_{\text{total}} = L\,i}$$

$$\boxed{\mathcal{E} = -L\frac{\mathrm{d}i}{\mathrm{d}t}}$$



SI unit of L : henry (H)

Self induction of a coil is said to be one henry if a current of 1A in the coil links a flux of 1 weber in it

Or

Self induction of a coil is said to be one henry if current changing at a rate of 1As^{-1} results in an induced emf of 1V in it

Energy stored in a solenoid

Consider a solenoid of self inductance L. As the current in the solenoid increases the magnetic field in it also increases. Induced emf that tends to oppose this change is

$$\mathcal{E} = -L\frac{\mathrm{d}i}{\mathrm{d}t}$$

Power delivered by external source to set up current in the inductor, against this induced emf, is given by

$$P = L\frac{\mathrm{d}i}{\mathrm{d}t}i$$

$$\frac{\mathrm{d}W}{\mathrm{d}t} = L\frac{\mathrm{d}i}{\mathrm{d}t}i$$

$$\mathrm{d}W = Li\,\mathrm{d}i$$

Integrating the above equation we get the total work done on the system

$$W = \int_0^i Li\,\mathrm{d}i$$

$$W = L\int_0^i i\,\mathrm{d}i$$

$$W = \frac{1}{2}Li^2$$

This work is stored as potential energy, therefore

$$\boxed{U = \frac{1}{2}Li^2}$$

Energy density in a solenoid

Consider a solenoid of length l. and area of cross-section A. In steady state condition, let the magnetic field set up in the solenoid be B.

Self inductance of the solenoid is given by

$$L = \mu_o n^2 l \pi r^2$$

Energy stored in the solenoid is

$$U = \frac{1}{2} L i^2$$

Using eq (i) we get

$$U = \frac{1}{2} \mu_o n^2 l \pi r^2 i^2$$

Energy per unit volume is

$$U_{\text{density}} = \frac{1}{2} \frac{\mu_o n^2 l \pi r^2 i^2}{A l}$$

$$U_{\text{density}} = \frac{1}{2} \mu_o n^2 i^2$$

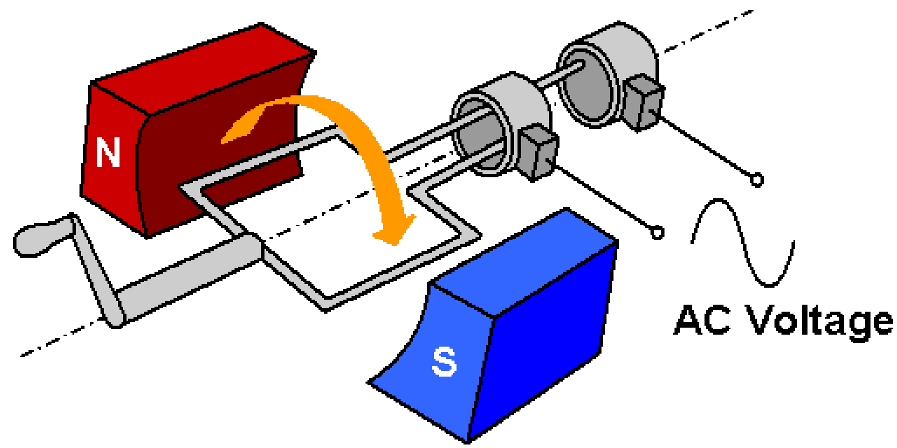
Magnetic field in a solenoid is given by $\mu_o n i$ therefore

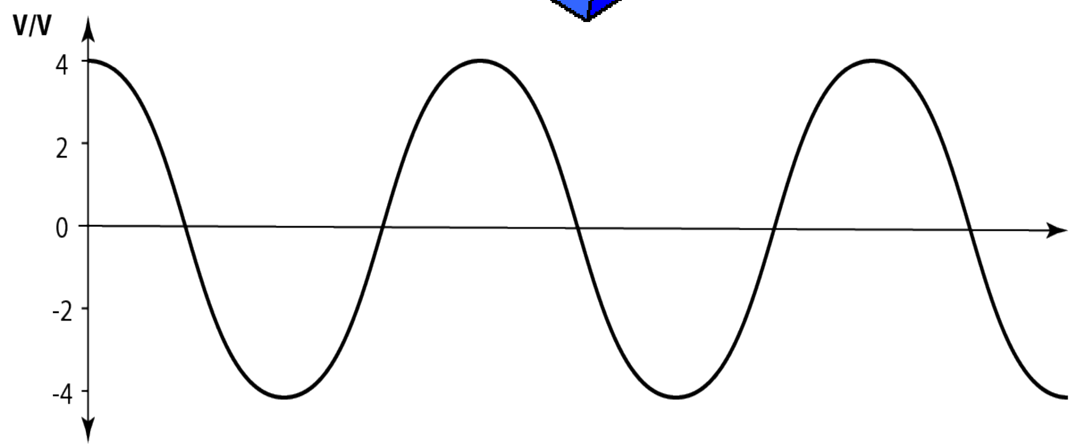
$$\boxed{U_{\text{density}} = \frac{B^2}{2\,\mu_o}}$$

a.c. generator

Principle : Induced emf in a coil due to electromagnetic induction as the coil is made to rotate in a region of magnetic field.

In the basic set up of an ac generator, a coil (also called armature) mounted on a rotor shaft. The ends of the coil are connected to an external circuit by means of slip rings and brushes. Axis of rotation of the coil is perpendicular to the direction of the magnetic field.





The coil is mechanically rotated in the region of uniform magnetic field by some external means. As the coil rotates, the magnetic flux associated with it changes. This changing magnetic flux results in an emf induced in the coil. An ac generator converts mechanical energy into electrical energy.

Electromagnetic induction

a.c. generator

Flux associated with the coil is given by

$$\phi = NBA\cos(\omega t)$$

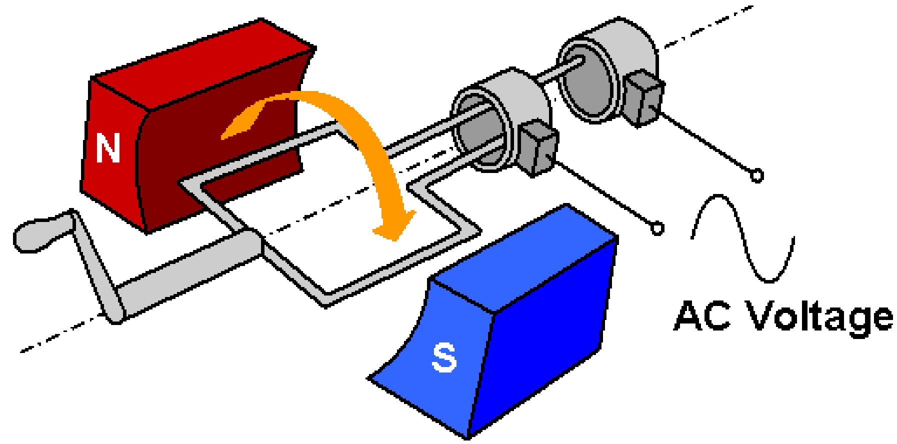
Induced emf is given by

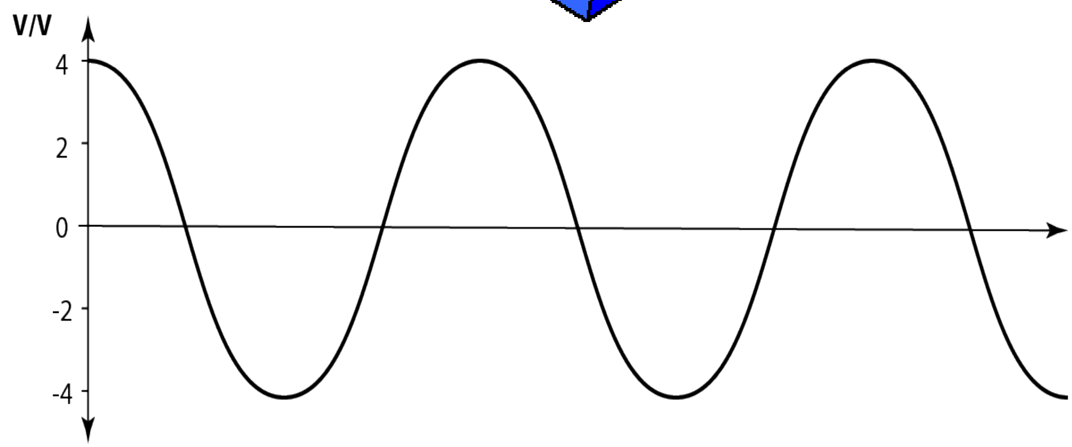
$$\mathcal{E} = -\frac{d\phi}{dt}$$

$$\mathcal{E} = -NBA\frac{d\cos(\omega t)}{dt}$$

$$\mathcal{E} = NBA\omega\sin(\omega t)$$

$$\mathcal{E} = \mathcal{E}_o\sin(\omega t)$$





Note : The mechanical energy required for rotation of the armature is provided by (a) falling water in *hydro-electric generators* (b) burning coal in *thermal generators* (c) nuclear fusion in *nuclear power generators*

Eddy currents

These are the currents induced in bulk conductors when they are subjected to changing magnetic field.

Eddy currents are induced due to changing magnetic flux which in turn is due to changing magnetic field.

Advantages:

(a) Magnetic breaking in electric trains

(b) damping of oscillations in galvanometer

(c) Heat generated in induction furnace

(d) Electric power meters

Disadvantage : Eddy currents lead to dissipation of energy in the form of heat.

Cores of transformers are laminated to reduce dissipation of heat due to eddy currents.